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REPORT

MRL-R-892

MANEUVERABILITY OF TORPEDOES IN THE HORIZONTAL PLANE

E.H. van Leeuwen

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POSTAL ADDRESS: Director, Materials Research Laboratories
P.O. Box 50, Ascot Vale, Victoria 3032, Australia

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AUTHOR(S)

van LEEUWEN, E.H.

CORPORATE AUTHOR

Materials Research Laboratories
P.O. Box 50,
Ascot Vale, Victoria 3032

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CONTENTS

MANEUVERABILITY OF TORPEDOES IN THE HORIZONTAL PLANE

1. INTRODUCTION

The objective of this paper is to model mathematically, aspects relating to the maneuverability of a modified torpedo. Of particular interest are the analytical expressions for the trajectory of a dynamically stable torpedo executing maneuvers in the horizontal plane. It will be assumed here, that different trajectories result as a consequence of different rudder positions. For example if the rudders are set in either the port or starboard position the resulting trajectory is a circle, whereas if the rudders are set in a neutral position the trajectory is a straight line.

The equations of the trajectory can be calculated in terms of the hydrodynamic coefficioients of the torpedo as will be shown in Section 3, using the equations of motion derived in [1]. In deriving the trajectory equations we are not concerned with the operation of the steering mechanism and to this end it is assumed that commands to turn the torpedo are carried out instantaneously with no lag time.

Finally in section 4 the turning circle of a torpedo which is rolled through an angle θ degrees is calculated in terms of the hydrodynamic coefficients of the torpedo.

2. EQUATIONS OF MOTION IN THE HORIZONTAL PLANE

The angular equations of motion for a torpedo moving along the x-axis in the horizontal plane through an ideal quiescent fluid can be expressed in a non-rolling coordinate frame in the ζ domain as [1]:

$$-C_{L_\psi}\psi + (C_{F_t} - m_L^*)\phi' + C_{L_\delta}\delta - A_{26}^*\phi'' = -m_T^*\psi' , \qquad (2.1a)$$

and

$$C_M \psi + C_{M_d}\varphi' + A^*_{26}\psi' + C_{M_\delta}\delta = \Theta^*\phi" \quad , \tag{2.1b}$$

where nomenclature is that of a previous paper [1] and defined in section 6. In (2.1) the time coordinate t has been replaced by a non-dimensional arc length coordinate ζ , with ( )' denoting differentiation with respect to ζ . The coordinate ζ is defined by:

$$\zeta := tt_c^{-1} \quad ,$$

where $t_c := V^{-1}\ell$ is called the characteristic time, ℓ is the length of the torpedo and V its velocity.

3. MANEUVERABILITY IN A NON-ROLLING COORDINATE FRAME

Aspects of maneuverability which are of interest here are (a) the turning radius of a torpedo corresponding to the path aa' in figure 3.1 when the rudders are locked in either the port or starboard position and (b) the equation of the path a'a" corresponding to the case where the rudders are set in a neutral position.

When the trajectory is an arc the radius of the turn maneuver R, maybe found by solving (2.1) with δ constant (i.e., δ=k). Similarly if the trajectory a'a" is to be found, then solutions to (2.1) are sought with δ = O.

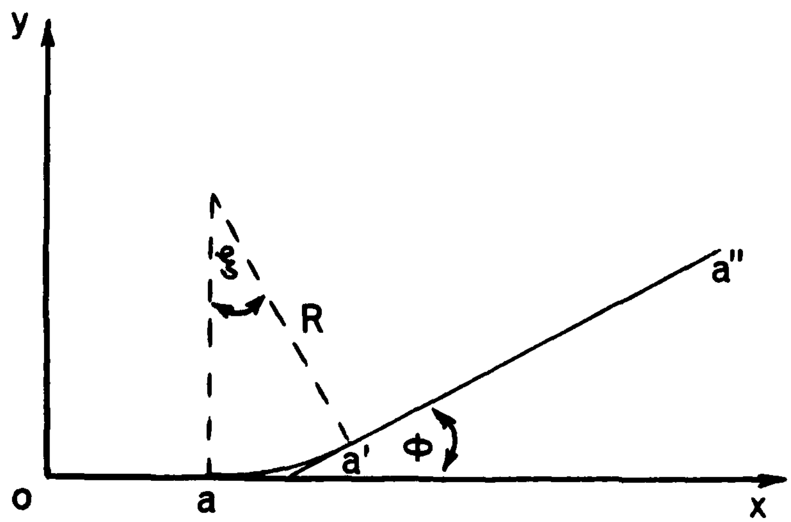

Figure 3.1. Torpedo trajectory in the horizontal plane.

Case (a) Calculation of R.

In order to calculate the equation of the trajectory aa' and the radius of curvature R, the angles ϕ and ψ need to be derived as a function of the independent variable ζ. ϕ can be found on decoupling (2.1a) and (2.1b) and yields an ordinary differential equation with constant coefficients:

$$\alpha_4\phi'''(\zeta) + \alpha_3\phi''(\zeta) + \alpha_2\phi'(\zeta) + \alpha_0 k = 0 \quad , \tag{3.1}$$

where

$$\alpha_4 = m_T^* \theta^* - A_{26}^{*\,2} \quad ,$$

$$\alpha_3 = - m_T^* C_{M_d} + A_{26}^*(C_{F_t} - m_L^*) - C_{L_\psi}\theta^* - C_M A_{26}^* \ , \tag{3.2}$$

$$\alpha_2 = -(m_L^* - C_{F_t})C_M + C_{L_\psi}C_{M_d} \ ,$$

$$\alpha_0 = C_M C_{L_\delta} + C_{L_\psi}C_{M_\delta} \quad .$$

Applying the Laplace transform method to (3.1) gives:

$$\overline{\varphi}(s) = \frac{s^3\varphi_0\alpha_4 + s^2(\alpha_4\varphi_0' + \alpha_3\varphi_0) + s(\alpha_4\varphi_0'' + \alpha_3\varphi_0' + \varphi_0\alpha_2) - \alpha_0 k}{s^2(\alpha_4 s^2 + \alpha_3 s + \alpha_2)} \ , \tag{3.3}$$

where $\phi(s)$ is the Laplace transform of the unknown function $\phi(t)$. The initial conditions $\phi_0(0)$, $\phi_0'(0)$ and $\phi_0''(0)$ completely determine the second term of (3.1). For a torpedo moving from a straight and level trajectory to a circular trajectory the initial conditions are $\psi = \phi_0 = \phi_0' = 0$ and from (2.1)

$$\varphi_0''(0) = \frac{k(m_T^* C_{M_\delta} - C_{L_\delta} A_{26}^*)}{m_T^*\Theta^* - A_{26}^{*2}} .$$

Performing a partial fraction expansion on (3.3) and then taking the inverse Laplace transform yields:

$$\varphi(\zeta) = \frac{T_3}{a_+a_-} + \frac{T_4(a_+ + a_-)}{(a_+a_-)^2} + \frac{T_4\zeta}{a_+a_-}$$
$$+ \frac{(T_1a_+^3 - T_2a_+^2 + T_3a_+ - T_4)\, e^{-a_+\zeta}}{a_+^2(a_+ - a_-)} \tag{3.4}$$

$$+ \frac{(-T_1 a_-^3 + T_2 a_-^2 - T_3 a_- + T_4)\, e^{-a_-\zeta}}{a_-^2(a_+ - a_-)} \quad ,$$

where

$$\begin{aligned} T_1 &:= \psi_0, \\ T_2 &:= \phi_0' + \alpha_3 \phi_0 \alpha_4^{-1}, \\ T_3 &:= \phi_0'' + (\alpha_3 \phi_0' + \phi_0 \alpha_2)\alpha_4^{-1}, \\ T_4 &:= -\alpha_0 k \alpha_4^{-1}, \end{aligned} \tag{3.5}$$

and a_+ and a_- are the roots of the quadratic:

$$\alpha_4 s^2 + \alpha_3 s + \alpha_2 = 0 \quad ,$$

that is

$$-a_\pm = - \frac{\alpha_3 \pm (\alpha_3^2 - 4\alpha_2\alpha_4)^{1/2}}{2\alpha_4} \ .$$

The angle ψ can be found by eliminating ψ' from (2.1a) and (2.1b) giving

$$\psi(\zeta) = T_5\{\alpha_4\phi'' - T_6\phi' - T_7\delta\}, \tag{3.6}$$

where

$$\begin{aligned} T_5 &:= (A_{26}^* C_{L_\psi} - m_L^* C_{M_d})^{-1} \quad , \\ T_6 &:= (m_T^* C_{M_d} - A_{26}^*(C_{F_t} - m_L^*)) \quad , \\ T_7 &:= (-A_{26}^* C_{L_\delta} + m_T^* C_{M_\delta}) \quad . \end{aligned} \tag{3.8}$$

Differentiating (3.4) and substituting ϕ' and ϕ'' into (3.6) yields:

$$\psi(\zeta) = \frac{\{T_1 a_+^3 - T_2 a_+^2 + T_3 a_+ - T_4)(a_+\alpha_4 + T_6)\, e^{-a_+\zeta}}{a_+(a_+ - a_-)}$$

$$+ \frac{(-T_1 a_-^3 + T_2 a_-^2 - T_3 a_- + T_4)(a_- \alpha_4 + T_6)\ e^{-a_- \zeta}}{a_-(a_+ - a_-)}$$

$$+ \left(\frac{T_6 \alpha_o}{a_+ a_-} - T_7\right) k \} T_5, \qquad (3.7)$$

In figure 3.1 the angle $\xi=\psi+\phi$ is simply the sum of (3.4) and (3.7). The first two terms of (3.7) and the last two terms of (3.4) are transients and die away exponentially as ζ increases. For large values of ζ, the dominant terms are:

$$\xi \sim \frac{T_3}{a_+ a_-} + \frac{T_4(a_+ + a_-)}{(a_+ a_-)^2} + k\left(\frac{T_6 \alpha_o}{a_+ a_-} - T_7\right) + \frac{T_4 \zeta}{a_+ a_-}\ , \qquad (3.9)$$

and is called the steady state solution. Under the coordinate transformation:

$$\zeta^* = \zeta + \left(\frac{T_3}{a_+ a_-} + \frac{T_4(a_+ - a_-)}{(a_+ a_-)^2} + k\left(\frac{T_6 \alpha_o}{a_+ a_-} - \Upsilon_7\right)\right)\frac{a_+ a_-}{T_4}\ ,$$

(3.9) may be written as:

$$\xi = \frac{T_4 \zeta^*}{a_+ a_-}\ . \qquad (3.10)$$

It follows from the above equation that the trajectory of the torpedo is a circle with radius

$$R = a_+ a_- T_4^{-1}\ ,$$

which in terms of the hydrodynamic coefficients and the rudder angle k can be expressed as:

$$R = \frac{(m_L^* - C_{F_t})C_M - C_{M_d} C_{L_\psi}}{(C_M C_{L_\delta} + C_{L_\psi} C_{M_\delta})k}\ . \qquad (3.11)$$

From (3.11) it follows that the radius R is inversely proportional to k if the composite change in the hydrodynamic coefficients is small, compared to changes in k.

Case (b) Equation of the Trajectory a'a''

The equation of the trajectory with neutral rudder response (ie., k=0) corresponding to the path a'a" in figure 3.1 can be found if ξ is small. This is achieved as follows. If $d\zeta$ is a differential element of arc along the trajectory then,

$$D_\zeta x = \cos\xi \text{ and } D_\zeta y = \sin\xi \; . \tag{3.12}$$

Assuming ξ to be small and integrating (3.12) gives

$$\int_a^x du = \int_0^\zeta dv \text{ or } x - a = \zeta, \tag{3.13a}$$

and

$$y = \int_0^\zeta \xi(\rho)\big|_{k=0} d\rho. \tag{3.13b}$$

The asymptotic form of (3.13b) for large ζ is a straight line given by:

$$y \sim \frac{\gamma_3 x}{a_+ a_-} + \frac{1}{(a_+ a_-)^2}\{T_3 a_+ a_-(1 + T_5 T_6) - T_3(a_+ + a_-)$$

$$+ a_+^2 a_-^2 (T_1 T_5 T_6 + \alpha_4 T_2 T_5 + T_1(a_+ + a_-))\} + \frac{\gamma_3 a}{a_+ a_-} \quad . \tag{3.14}$$

Thus it can be concluded if the rudder angle is set to the neutral position (ie., k=0) the torpedo comes out of the turn aa' and into a straight line a'a".

4. MANEUVERABILITY AND ROLL

Consider a dynamically stable torpedo moving rectilinearly along the x-axis with velocity V. If the torpedo rolls through an angle of θ degrees, when turning commences, then the equations governing the motion are similar to (2.1) but with the terms $C_{L_\delta}\delta$ and $C_{M_\delta}\delta$ replaced by:

$$C_{L_\delta}\delta = C_{L_{\delta r}}\delta r \cos\theta - C_{L_{\delta e}}\delta e \sin\theta \quad ,$$

and

$$C_{M_\delta}\delta = -C_{M_{\delta e}}\delta e \sin\theta + C_{M_{\delta r}}\delta r \cos\theta \quad ,$$

where δe and δr are the elevator and rudder angles respectively. Applying the procedure outlined in section 3, the decoupled equation for ϕ is similar to (3.1) but with the term $\alpha_o k$ replaced by:

$$(C_M C_{L_{\delta r}} + C_{L_\psi} C_{M_{\delta r}})\delta r \cos\theta - (C_{L_\psi} C_{M_{\delta e}} + C_{L_{\delta e}} C_M)\delta e \sin\theta. \qquad (4.1)$$

Assuming (4.1) is independent of ζ, the asymptotic form of ξ is again found to be:

$$\xi = \frac{T_4 \zeta^*}{a_+ a_-} \quad ,$$

where radius of the trajectory aa' is now:

$$R = \frac{-C_{L_\psi} C_M + (m_L^* - C_{F_t}) C_{M_d}}{(C_M C_{L_{\delta r}} + C_{L_\psi} C_{M_{\delta r}})\delta r \cos\phi - (C_{L_\psi} C_{M_{\delta e}} + C_M C_{L_{\delta e}})\delta e \sin\phi} \; , \qquad (4.2)$$

which reduces to (3.11) if $\theta = 0$ and $\delta r = k$. If the degree of roll does not become large, we can put $\sin\theta \simeq \theta$ and $\cos\theta \simeq 1$ in (4.2). Assuming the ratio

$$\frac{C_{L_\psi} C_{M_{\delta e}} + C_M C_{L_{\delta e}}}{C_M C_{L_{\delta r}} + C_{L_\psi} C_{M_{\delta r}}} \; ,$$

is constant for small θ, then (4.2) takes the form:

$$R \simeq R_o \left[1 - \frac{(C_{L_\psi} C_{M_{\delta e}} + C_M C_{L_{\delta e}})}{(C_M C_{L_{\delta r}} + C_{L_\psi} C_{M_{\delta r}})} \frac{\delta e}{\delta r} \theta\right]$$

where R_o is the radius of the turning circle in the absence of roll and is given by (3.11) with $k = \delta r$.

5. DISCUSSION

The treatment of maneuverability discussed in this paper is idealized since consideration is given only to the hydrodynamic aspects of turning. It has been assumed that when the rudder of the torpedo is locked in a particular position, a hydrodynamic pressure is created which is sufficient to enable a turn to commence immediately. The "lag" time is taken to be zero. Under these assumptions the trajectory of a torpedo in the horizontal plane with rudders fixed can be calculated and the results are presented in section 3. In section 4 the radius of a turn has been derived with the torpedo rolled through an angle of θ degrees. Provided the hydrodynamic coefficients of the torpedo are known, the radii (3.11) or (4.2) can be explicitly found.

6. NOMENCLATURE

C_{L_ψ} Lift coefficient (horizontal plane) ,

C_{F_t} Transverse force coefficient ,

C_{L_δ} Rudder lift coefficient ,

C_{M_δ} Rudder moment coefficient ,

C_{M_d} Damping moment coefficient corrected for entrained water (horizontal plane) ,

C_M Moment coefficient corrected for entrained water ,

A^*_{26} Component of the added mass tensor ,

m^*_T Longitudinal mass due to entrained water ,

m^*_L Transverse mass due to entrained water ,

Θ^* Moment of inertia corrected for entrained water (horizontal plane) ,

δ Rudder angle,

R Radius of curvature of turn maneuver,

ϕ Orientation angle in the horizontal plane (yaw angle),

ψ Angle of attack in yaw,

7. REFERENCES

1. Van Leeuwen, E.H., 1982 "A study of the Motion and Stability of Torpedoes in 3 Degrees of Freedom", MRL-R-891.

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